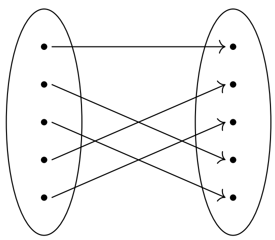
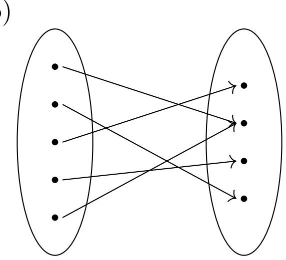
MATH 120A Prep: Functions

1. Each of the following is a visual representation of a function. It's domain and codomain are represented by points in a circle and the arrows between them describe how elements in the domain map to elements in the codomain. For each function determine whether or not it is a function, and if so whether it is injective, surjective, bijective, or none of the above.

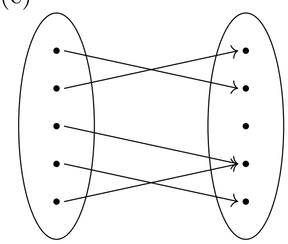
(a)



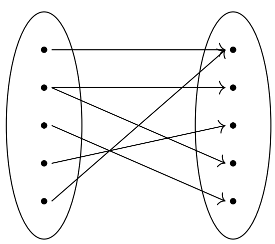
(b)



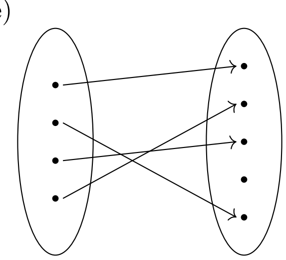
(c)



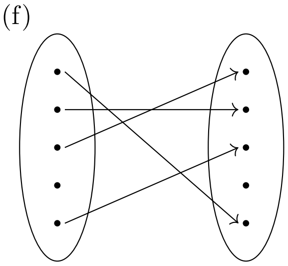
(d)



(e)



(f)



Solution: (a) Bijective (b) Surjective (c) Function (d) Not function (e) Injective (f) Not function

2. Determine whether the function $f : \mathbb{R} \to \mathbb{R}^2$ where $f(x) = (x^2, -2x)$ is injective and/or surjective.

Solution: This map is injective but not surjective.

Injective: Suppose $f(x) = f(y)$, so $(x^2, -2x) = (y^2, -2y)$ which means $x^2 = y^2$ and $-2x = -2y$. But then the second equation implies $x = y$.

Not surjective: The first coordinate of the output is always positive so this can't be surjective, for example $(-1, 0)$ is not equal to $f(x)$ for any x.

3. Let $S = \{(x, y) \in \mathbb{R}^2 : x^2 + y^2 = 1\}$ and define a function $g : S \to [-1, 1]$ by $g(x, y) = x$. Determine whether this map is injective and/or surjective.

Solution: This map is not injective but it surjective.

Not injective: $(0, 1) \neq (0, -1)$ but $g(0, 1) = 0 = g(0, -1)$.

Surjective: Let $a \in [-1, 1]$, we want a point (x, y) so $g(x, y) = a$. Since $g(x, y) = x$, we need $x = a$. Now we need a corresponding y value so $x^2 + y^2 = a^2 + y^2 = 1$. Since $a \in [-1, 1]$, $a^2 \in [0, 1]$ and so $1 - a^2 \geq 0$. Thus we can take a square root, so let $y = \sqrt{1 - a^2}$. Then $a^2 + \sqrt{1 - a^2}^2 = 1$ so $(a, \sqrt{1 - a^2}) \in S$ and $g(a, \sqrt{1 - a^2}) = a$ as desired.